Sequential R&D and Blocking Patents in the Dynamics of Growth

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Abstract

The incentives to conduct basic or applied research play a central role for economic growth. How does increasing early innovation appropriability affect basic research, applied research, innovation and growth? In a common law system an explictly dynamic macroeconomic analysis is appropriate.

This paper analyzes the macroeconomic effects of patent protection by incorporating a two-stage cumulative innovation structure into a quality-ladder growth model with skill acquisition. We focus on two issues: (a) the over-protection vs. the under-protection of intellectual property rights in basic research; (b) the evolution of jurisprudence shaping the bargaining power of the upstream innovators. We show that the dynamic general equilibrium interations may seriously mislead the empirical assessment of the growth effects of IPR policy: stronger protection of upstream innovation always looks bad in the short- and possibly medium-run. We also provide a simple "rule of thumb" indicator of the basic researcher bargaining power.

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1 Introduction

Is an increase in the intellectual protection of basic research beneficial or harmful[1] for innovation and growth? It is well known that the US economy in the 1980s witnessed a strengthening of intellectual property rights (IPR). This implied an increase in the relative bargaining power of the upstream innovators, and therefore a decrease in the relative bargaining power of the downstream innovators (i.e. applied researchers or developers). This paper will show that this has harmful short-run consequences for economic growth, even though it could be conducive to higher growth in the longer term. In fact, being basic and applied research endogenous, they respond to the underlying relative bargaining power of the upstream innovators in two opposite ways: R&D reallocates more upstream, thereby slowing down the pace of innovation for a while. As a consequence, econometric studies may end up wrongly detecting negative short-term effects of basic research on innovation and growth, with potentially misleading research policy implications. Therefore, by focussing on the composition effects of patent strength, we can explain the weak effects of stronger IPR on total R&D and on growth, often found in the data. This analysis is related to the existing literature on basic research and economic growth, such as Gersbach, Schneider, and Schneller, (2010a and b), and Spinesi (2007 and 2012), however here the focus is on the evolution of the private incentives for basic research by universities or other institutions.

We will cast our analysis in a dynamic general equilibrium framework, to better capture all innovation-related features of the economy. This necessarily includes endogenous skill acquisition by individuals who differ in their abilities, as in Dinopoulos and Segerstrom (1999). When properly taken into account, we shall see that the normative assignment of relative bargaining power of basic versus applied researcher is also related to the returns to education. However, in our analysis, tracking the dynamics of the skill premium and endogenizing education is only instrumental to a better screening of the short- and long-term effects of basic research incentives on economic growth. A theory of functional income inequality should instead consider that R&D employment is only a fraction of the total employment of college workers, as

[1] Heller and Eisemberg (1998) suggested the existence of a *tragedy of the anticommons*, i.e. a proliferation of upstream intellectual property rights which greatly amplify the transaction cost of downstream research and development, thus hampering downstream research for biomedical advance.

in Galor and Moav's (2000) model in which skills are endogenous, and the demand for schooling is increasing with ability-biased technological change; and as in Acemoglu (1998 and 2002) and Kiley (1999), which show that education increases the market for the skill complementary inputs, thereby driving up the profitability of innovations that increase the productivity of the skilled and therefore the returns to higher education.

In the microstructure of our model, a two-stage cumulative innovation structure is developed: unlike Grossman and Shapiro (1987) and Green and Scotchmer (1995) we consider free entry by a multitude of firms. Differently from Bessen and Maskin's (2009) analysis of sequential R&D with complementary innovation, our approach features creative destruction. Unlike Chu's (2010) unambiguous effect of general IPR strengthening on inequality, we will show that tightening patent protection in basic research may increase or decrease wage inequality. Similarly to Furukawa (2007), increasing upstream patent rights has an inverted U-shaped effect on growth, but due to restricted development rather than reduced experience.

In our framework, basic and applied research technologies are heterogenous and the bargaining power of the upstream innovation changes[2], thus stylizing the evolution of the US jurisprudence after 1980. From that date on, the US national system of innovation has been re-shaped by a sequence of important new laws and by a cumulative sequence of sentences that set the precedents for future modifications in the jurisprudence. All these changes pointed to an increase in the appropriability of innovations at their initial stages[3]. Being the US legal system a common-law regime, the jurisprudence evolved gradually[4] in the direction of stricter intellectual protection of re-

[2]Our framework somewhat complements Eicher and García-Peñalosa, (2008), that envisages endogenous IPR based on firm choice, instead of on jurisprudence evolution.

[3]Including the Stevenson-Wydler act of 1980 and the Bayh-Dole act, of 1980, amended the patent law, to facilitate the commercialization of inventions obtained thanks to government funding, especially by universities. The pro-early innovation cultural change is also reflected in the increasing protection of trade-secrets - starting in the 80s with the Uniform Trade Secret Act and culminating with the Economic Espionage Act of 1996 (Cozzi, 2001) - as well as in the increasingly positive attitude towards software patents (Hunt, 2001, Hall, 2009), culminating in the Final Computer Related Examination Guidelines issued by the *USPTO* in 1996.

[4]In our case, it is important to recall Janice Mueller's (2004) account of the common law development of a narrow experimental use exemption from patent infringement liability: with special reference to the discussion of the change in the doctrine from 1976's *Pitcairn v. United States*, through 1984's Federal Circuit decision of *Roche Products, Inc. v.Bolar*

search tools[5], basic research ideas[6], etc. The essence of the common law is that it is made by judges sitting in courts, by applying their common sense and knowledge of legal precedent (*stare decisis*) to the facts before them. During the early 1980s began a progressive process in which the U.S. Court decisions changed from the old doctrine limiting the patentability of early-stage scientific discoveries to the conception that also fundamental basic scientific findings (such as genetic engineering procedures or semiconductor designs) are patentable. This process took a quarter century, culminating in the 2002 *Madey vs. Duke University* Federal Circuit's decision, which completed a process of elimination of the "research exemption" to patent claims. Interestingly, the more recent cases seem to be witnessing an opposite trend, most notably *Merck vs Integra Lifesciences* (2005), in which the Supreme Court decided to re-affirm research exemption in the pharmaceutical sector.

If what deeply characterizes common law (and sharply separates it from the Continental Europe type legal systems) is an uninterrupted continuity such that within the *stare decisis* regime an institutional break point is even hardly conceivable, we must conclude that the analysis of the effects of the US patent policy on the economy is forced to include the whole transition dynamics. The law and economics literature is currently modelling the evolution of the case law in the perspective of analyzing Benjamin Cardozo's and Richard Posner's view of common law as efficiency promoting. In fact, according to this influential view, unlike civil law, being the common law decentralized, it follows the aggregate decision making of several heterogenous judges, whose idiosyncratic opinions average one another. Moreover, the very sequential precedent structure, implies that (Gennaioli and Shleifer, 2007b) one appellate court overrules another's decision, tending to progressive mitigation and efficiency only if the majority of the judges is unbiased, depending also on the judge's effort cost of changing the legal rule established in a precedent. Appellate courts may change a previously established legal rule also by "distinguishing" the case based on the consideration of a "previously neglected dimension" (Gennaioli and Shleifer, 2007a), which can facilitate convergence towards a more efficient legal rule. We inquire on whether the increasingly pro-upstream R&D court orientation from 1980 to 2002 has been following an improvement in promoting innovation or if it has

Pharmaceutical Co., all the way to *Madey v. Duke University* in 2002.

[5]Another important source of change in sharpening IPRs can be driven by special interests, as studied by Chu (2008).

[6]See Gallini (2002), Mueller (2002 and 2004), Scotchmer (2004).

ended up following the bias of less and less liberal judges. In this paper, we look for potentially detectable aspects of the time series of several important variables - skill wage premium, education, innovation, labour force allocation, and the market value of patents - associated with either long-term evolution of the legal rules. In doing so, we follow a dynamic general equilibrium perspective, which allows us to assume that economic agents are sufficiently intelligent to detect what "trend" is occurring, and suitably take optimizing decisions.

In order to analyze the effects of an expected and progressive change in the patent protection of basic research, we therefore need to simulate all variables in their transitional dynamics. We will extract lessons from our numerical results, useful to detect whether increasing basic research protection common law doctrine is gradually facilitating the national system of innovation or evolving for the worse.

The remainder of this paper is organized as follows. Section 2 and Section 3 set the model and Section 4 characterizes the equilibrium. Section 5 analyzes a simple special case, useful as a benchmark. Section 6 identifies an important problem with blocking patents. In Section 7 we show the numerical simulations. Section 8 concludes. The most challenging proofs are in the Appendix.

2 The Model

2.1 Households

We assume a large number of dynastic families - normalized to 1 for simplicity - whose members, born at birth rate b and passing away at rate δ, live a period of duration D. The resulting population growth rate[7] is $g = b - \delta > 0$. This demographic structure implies the following restrictions: $b = \frac{ge^{gD}}{e^{gD}-1}$ and $\delta = \frac{g}{e^{gD}-1}$.

At time t the total number of individuals is e^{gt}. Each individual can spend her life working as unskilled or studying the first $T_r < D$ periods and

[7] Dinopoulos and Segerstrom (1999) have first developed the overlapping generations education framework followed here. Boucekkine et al. (2002) and Boucekkine et al. (2007) recently studied population and human capital dynamics in continuous time and off steady states and numerically calibrated in a way methodologically more similar to ours.

then working as skilled. Each individual cares only about the utility of the average family member. Hence, despite bounded individual life, the individual decisions are taken within the household by maximizing the following intertemporally additive utility functional:

$$U = \int_0^{\infty} e^{-\rho t} u(t)\, dt, \tag{1}$$

where $\rho > 0$ is the subjective rate of time preference. Per-family member instantaneous utility $u(t)$ is defined as:

$$u(t) = \int_0^1 \ln \left[\sum_j \gamma^j d_{jt}(\omega) \right] d\omega, \tag{2}$$

where $d_{jt}(\omega)$ is the individual consumption of a good of quality $j = 1, 2, ...$ and produced in industry ω at time t, and bought at price $p_{jt}(\omega)$. Parameter $\gamma > 1$ measures the size of the quality upgrades.

Defining per-capita expenditure on consumption goods as $E(t) = \int_0^1 \left[\sum_j p_{jt}(\omega)\, d_{jt}(\omega) \right] d\omega$, the real interest rate as $i(t)$, and time 0 family wealth as $A(0)$, the intertemporal budget constraint is $\int_0^{\infty} e^{gt - \int_0^t i(\tau) d\tau} E(t)\, dt \leq A(0)$.

Following standard steps of quality ladders models[8], the consumers will only buy good with the lowest quality adjusted price, and the Euler equation follows:

$$\dot{E}(t)/E(t) = i(t) - (\rho + g) = r(t) - \rho, \tag{3}$$

where $r(t) \equiv i(t) - g$ is the population growth deflated instantaneous market interest rate at time t, and, together with the transversality condition, determines consumer choice.

Individuals differ in their learning ability θ, which, for each generation, is uniformly distributed over the unit interval. Hence an individual of ability $\theta \in [0, 1]$ will be able to acquire $\theta - \Gamma$ units of human capital after an indivisible training period of length T_r. The only cost of education is the individual's time, which prevents her from earning the unskilled wage w_u. In what follows we choose unskilled labour as our numeraire, and therefore set $w_u(t) = 1$ at all $t \geq 0$.

[8]See Segerstrom et al. (1990), Grossman and Helpman (1991) and Segerstrom (1998).

Hence an individual born at t with (known) ability $\theta(t) \in [0,1]$ and who decides to educate herself will earn nothing from t to $t+T_r$, and then earn a skilled wage flow $(\theta(t)-\Gamma)w_H(s)$ at all dates $s \in [t+T_r, t+D]$, which implies that at time t there will exist an ability threshold $\theta_0(t) \in [\Gamma, 1]$ below which the individual decides to work as an unskilled. Threshold $\theta_0(t)$ solves the following equation:

$$\int_t^{t+D} e^{-\int_t^s i(\tau)d\tau} ds = (\theta_0(t) - \Gamma) \int_{t+Tr}^{t+D} e^{-\int_t^s i(\tau)d\tau} w_H(s) ds,$$

obtaining

$$\theta_0(t) = \Gamma + \frac{\int_t^{t+D} e^{-\int_t^s i(\tau)d\tau} ds}{\int_{t+Tr}^{t+D} e^{-\int_t^s i(\tau)d\tau} w_H(s) ds}. \tag{4}$$

Since in a steady state $i(t) = \rho + g$, the steady state level of $\theta_0(t)$ is

$$\theta_0 = \Gamma + \frac{1 - e^{-(\rho+g)D}}{\left[e^{-(\rho+g)Tr} - e^{-(\rho+g)D}\right] w_H}, \tag{5}$$

where w_H denotes the steady state skill premium.

2.2 Manufacturing

In each final good industry $\omega \in [0,1]$ and for each quality level $j(\omega)$ of the good, production is carried out according to the following Cobb-Douglas technology

$$y(\omega, t) = X^{\alpha}(\omega, t) M^{1-\alpha}(\omega, t), \text{ for all } \omega \in [0,1], \tag{6}$$

where $\alpha \in (0,1)$, $y(\omega, t)$ is the output flow at time t, $X(\omega, t)$ and $M(\omega, t)$ are the skilled and unskilled labour inputs. In each industry firms minimize costs by choosing input ratios

$$\frac{X(\omega)}{M(\omega)} = \frac{1}{w_H(t)} \frac{\alpha}{1-\alpha}. \tag{7}$$

The total per-capita amount L of unskilled labour only works in the manufacturing sectors. Therefore the aggregate skilled labour demand is equal to:

$$X(\omega,t)=\frac{1}{w_H(t)}\left(\frac{\alpha}{1-\alpha}\right)L(t)P(t) \tag{8}$$

In per-capita terms,

$$x(\omega,t)\equiv\frac{X(\omega,t)}{P(t)}=\frac{1}{w_H(t)}\left(\frac{\alpha}{1-\alpha}\right)L(t)\equiv x(t). \tag{9}$$

As in Aghion and Howitt (1992), skilled labour can also work in the R&D sectors. Therefore, a higher skill premium $w_H(t)$ frees resources for the R&D sectors.

We assume instantaneous Bertrand competition in all sectors. Since only the owner of the most recent top quality good patent can produce the top quality version of its sector good, the equilibrium price will be equal to a mark-up $\gamma>1$ over the unit cost $c(w_H(t),1)$. Moreover, as usual with Cobb-Douglas preferences, in a symmetric equilibrium per-capita demand is $d(t)=\frac{E}{\gamma c(w_H(t),1)}$. Therefore is each sector the temporary monopolist who owns the top quality product patent earns the same profit which, in per-capita terms, is equal to[9]:

$$\begin{aligned}\pi(t) &= \frac{\gamma-1}{\gamma}E(t)=(\gamma-1)\frac{w_H(t)x(t)}{\alpha}= \\ &= (\gamma-1)\frac{1}{1-\alpha}L(t). \end{aligned}\tag{10}$$

3 R&D and Innovation

The quality level j of each final product of variety $\omega\in[0,1]$ can increase as a result of R&D undertaken by private firms. In order to capture the

[9]The second equality builds on the Cobb-Douglas property that minimum total cost is $\left[\left(\frac{1-\alpha}{\alpha}\right)^{-(1-\alpha)}+\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\right]w_s^{\alpha}w_u^{1-\alpha}X^{\alpha}(\omega)\,M^{1-\alpha}(\omega)$. Hence profit is $(\gamma-1)$ times total cost. Using eq. (8) and simplifying gives the result.

interaction between basic and applied research, we assume that a basic research idea is a pre-requisite to applied research and applied R&D success opens the door for a further basic research advance. The first stage - basic research - of the product quality jump is the outcome of a Poisson process with probability intensity $\frac{\lambda_0}{P(t)}\left(\frac{N_B(\omega,t)}{P(t)}\right)^{-a}$ per unit of research labour, where $\lambda_0 > 0$ is a basic research productivity parameter, $N_B(\omega, t)$ is the mass of research labour employed in sector ω at time t, and $a > 0$ is a congestion externality parameter.

The second stage - applied research - completes the basic research idea and generates the new higher quality good according to a Poisson process with probability intensity $\frac{\lambda_1(t)}{P(t)}\left(\frac{N_A(\omega,t)}{P(t)}\right)^{-a}$ per unit of research labour, where $\lambda_1(t) > 0$ is an applied research productivity, viewed by the firms as a constant; $N_A(\omega, t)$ is the mass of research labour employed in sector ω at time t; and $a > 0$ is the congestions externality parameter. The presence of population size, $P(t)$, in the denominator states that R&D difficulty increases with the total population in the economy[10], which delivers endogenous growth without the strong scale effect[11], as suggested by Smulders and Van de Klundert (1995), Young (1998), Peretto (1998 and 1999), Dinopoulos and Thompson (1998), Howitt (1999), and recently confirmed empirically by Ha and Howitt (2006) and Madsen (2008).

Defining $n_B(\omega, t) \equiv \frac{N_B(\omega,t)}{P(t)}$ and $n_A(\omega, t) \equiv \frac{N_A(\omega,t)}{P(t)}$, as the skilled labor employment in each basic and, respectively, applied R&D sector, we can express the expected innovation rate in a ω' sector undertaking only basic R&D as $\lambda_0 n_B(\omega', t)^{1-a}$ and the expected innovation rate in a ω'' sector undertaking only applied R&D as $\lambda_1(t)n_A(\omega'', t)^{1-a}$. All stochastic processes are independent both across sectors and across firms. Hence, the existence of a continuum of sectors implies that the law of large number applies and aggregate variables evolve deterministically. Since all sectors switch from hosting only basic R&D firms - belonging to subset $A_0(t) \subset [0, 1]$ - to hosting only applied R&D - belonging to subset $A_1(t) \subset [0, 1]$ - the mass of sectors belonging to each type will flow deterministically[12]. Notice that $A_0(t) \cup A_1(t) = [0, 1]$ and $A_0(t) \cap A_1(t) = \emptyset$. Moreover, in our model, symmetric equilibria exist,

[10] Population density favours innovation at the local level (see Hunt, Chatterjee, and Carlino, 2001): according to this solution to the strong scale effect, the dilution of R&D is not related to population density, but with the overall size of the economy.

[11] See Dinopoulos and Thompson (1999) and Jones (2005).

[12] Provided the initial mass Lebesgue mass of each was positive.

allowing us to simplify notation: $n_B(\omega,t) \equiv n_B(t)$ and $n_A(\omega,t) \equiv n_A(t)$. Therefore, if $m(A_0(t)) \in]0,1[$ is the Lebesgue mass of the $A_0(t)$ subset - and hence $m(A_1(t)) = 1 - m(A_0(t))$ the Lebesgue mass of $A_1(t)$ subset - its evolution would be deterministic and described by the following first order differential equation:

$$\frac{dm(A_0(t))}{dt} = (1 - m(A_0(t)))\,\lambda_1(t)\,(n_A(t))^{1-a} - m(A_0(t))\lambda_0\,(n_B(t))^{1-a}\,. \quad (11)$$

The first term on the right hand side gives the flow of applied research results, arising in the fraction of sectors where basic research have been completed; the second term on the right hand side measures the flow of new basic research results. We also assume that the aggregate output of basic research increases the productivity of applied research: $\lambda_1(t) = \overline{\lambda}_1 \left(1 + \lambda_0 \left[\int_0^1 n_B(\omega,t)d\omega\right]^{1-a}\right)^{\varphi}$, where $\overline{\lambda}_1$ and φ are positive constants. This formulation introduces the possibility of cross-fertilization of applied research by other sector's basic research findings[13]. In symmetric equilibrium $\lambda_1(t) = \overline{\lambda}_1 \left(1 + \lambda_0 \left[n_B(t)\right]^{1-a}\right)^{\varphi}$.

We assume free entry into basic and applied research. Each inventor, be she basic or applied, is granted a patent. However, though the first R&D firm that invents a new final product gets the patent anyway, it will infringe the patent held by the previous basic research inventor. Therefore it will have to bargain with the basic research patent holder in order to produce the new version of this good.

Such a framework, incorporating to Green and Scotchmer (1995) research exemption regime for pure research tools[14], captures important aspects of the real world disputes between inventors whose patent claims allow the blocking of inventions[15]. Let $\beta(t) \in]0,1[$ denote the share of the final product (applied) patent value assigned - at the end of the negotiations taking place at time t - to the upstream (basic) patent holder[16]. This share captures time t

[13] This is complementary to Howitt's (1999) assumption of general knowledge, $A_t^{\max}$, being positively affected by the aggregate applied R&D.

[14] Also see Scotchmer (2004) and Nagaoka and Aoki (2006) for microeconomic analysis of this important case.

[15] O'Donoghue and Zweimueller (2004) and Chu (2009) are indirectly related, as they capture the role of patent claims in molding the bargaining between current and future innovators: their concepts of patentability requirement and leading breadth could be re-adapted here to accomodate the blocking power of the upstream patent holder.

[16] Assuming that basic and applied innovators matched and targeted applied innovator-

court orientation towards intellectual property. Changes in the jurisprudence towards stronger patent claims and weakening research exemptions would correspond to increases in $\beta(t)$, whereas a gradually looser upstream patent holder protection and stronger research exemptions would correspond to a declining $\beta(t)$. Taken literally, $\beta(t)$ can be obtained as a Nash bargaining solution between the patent blocker and the applied developer, with the courts orientation dictating the relative bargaining power. Theoretically, it is important to notice that both patent holders' outside options are zero: in case bargaining breaks down the applied patent holder cannot produce (zero profit), while the patent blocker cannot complete or find another completer because the application is now prior art[17].

In what follows, we will consider gradual changes in patent policy in terms of the sign of $\dot{\beta}(t)$. We will assume that the following specification holds:

$$\dot{\beta}(t) = (1 - \psi)(\bar{\beta} - \beta(t)). \tag{12}$$

Equation (12) is a linear differential equation with constant coefficient, which describes the speed of change in $\beta(t)$ per unit time. Parameter $\psi < 1$ guarantees asymptotic stability and $\bar{\beta} \in]0, 1[$ is the steady state. We will consider the progressive tightening of intellectual property rights in the US as the result of a sudden change in $\bar{\beta}$, which determines a gradual increase in $\beta(t)$ from its previous lower steady state level to its new level. It is important to notice that we are in a rational expectation framework: all economic agents after the regime change can predict the successive increases in $\beta(t)$, and the transition to a tight IPR regime is known to the agents from the beginning and all decisions are re-optimized. Hence all our numerical simulations are immune to Lucas' critique. In fact, the steady upstream shift of innovation incentives is too regular not to be incorporated in people's expectations, which leads law scholars to view 1980 as a sort of structural break of equation (12), and forces us to study the whole transitional dynamics of the model's

specific innovations, could re-read this strategic interaction as Aghion and Tirole's (1994a and b) research unit (RU) and customer (C). Then our case would clearly correspond to when RU's effort is important ($\tilde{U}_C > U_C$), which implies that "the property right is allocated to RU" (Aghion and Tirole, 1994b, p. 1191). In this light, our $\beta(t)$ generalizes Aghion and Tirole's (1994a and b) equal split assumption.

[17] In the realistic case that basic research results have multiple applications, this would increase the blocker's outside option and its equilibrium share of the final patent value.

economy. The statutory decisions taken in the early 1980 triggered a gradual change in the common law[18].

4 Equilibrium

Let us define v_B, v_L^0, and v_L^1 as the population-adjusted present expected value of a basic research patent (v_B), of an A_0 industry quality leader (v_L^0), and of an A_1 industry challenged leader (v_L^1).

Costless arbitrage between risk free activities and firms' equities imply that in equilibrium at each instant the following equations shall hold:

$$w_H(t) = \lambda_0 n_B(t)^{-a} v_B(t) \tag{13a}$$

$$r(t)v_B(t) = \lambda_1(t) n_A(t)^{1-a} \left(\beta(t) v_L^0(t) - v_B(t)\right) + \frac{dv_B(t)}{dt} \tag{13b}$$

$$w_H(t) = \lambda_1(t) n_A(t)^{-a} \left(1 - \beta(t)\right) v_L^0(t) \tag{13c}$$

$$r(t)v_L^0(t) = \pi(t) - \lambda_0 n_B(t)^{1-a} \left(v_L^0(t) - v_L^1(t)\right) + \frac{dv_L^0(t)}{dt} \tag{13d}$$

$$r(t)v_L^1(t) = \pi(t) - \lambda_1(t) n_A(t)^{1-a} v_L^1(t) + \frac{dv_L^1(t)}{dt} \tag{13e}$$

The value of a monopolist in an A_0 industry, v_L^0, has to obey equation (13d): in fact, the shareholders of the current quality leader compare the risk free income, rv_L^0, obtainable from selling their shares and buying risk free bonds to the expected value of their profits, π, net of probable capital loss, $\lambda_0 n_B^{1-a} \left(v_L^0 - v_L^1\right)$, in case a new basic research result appears in the industry.

As soon as a new basic R&D result appears in the industry, the incumbent monopolist's value falls down to a lower, but still positive, value v_L^1, which has to obey eq. (13e): as before, risk free income is equated to expected profits net of expected capital loss, but now the probability of the basic research idea's being completed by applied research in the industry, $\lambda_1 n_A^{1-a}$, is the monopolistic profit hazard rate, as the arrival of the new final product implies the complete displacement of the current leading edge product.

[18] According to Fon and Parisi (2006), such a case evolution could also appear in a civil law system.

Equation (13a) characterizes free entry into basic R&D (in an A_0 industry), equalizing the skilled wage to the probability $\lambda_0 n_B^{-a}$ of inventing times the value v_B of the resulting patent.

Equation (13b) equates the risk free income from selling a basic R&D patent, rv_B, to the expected present value of holding it in an A_1 industry. These expected increase in value deriving from someone else's - the n_A downstream researchers' - discovering the industrial application, of value v_L^0, plus the gradual appreciation in the case of someone else's R&D success not arriving, $\frac{dv_B}{dt}$.

Equation (13c) is the free entry condition for applied researchers that rationally expect to appropriate only fraction $1-\beta$ of the value of the final good monopolist.

As in the previous section, the industrial dynamics of this economy is described by equation (11):

$$\frac{dm(A_0(t))}{dt} = (1 - m(A_0(t)))\,\lambda_1(t)\,(n_A(t))^{1-a} - m(A_0(t))\lambda_0\,(n_B(t))^{1-a}\,. \quad (14)$$

These equations must be supplemented with the skilled labour market equilibrium condition

$$x(t) + m(A_0(t))n_B(t) + (1 - m(A_0(t)))\,n_A(t) = h(t), \quad (15)$$

where $h(t) \equiv H(t)/P(t)$ is the aggregate population-adjusted human capital.

5 Analysis of a Benchmark Special Case

The analytical results of this section are obtained under the assumption that $\rho = 0$; since all steady state equations are continuous in all variables and parameters, its comparative statics results continue to hold in a positive neighborhood where $\rho > 0$. Hence while the transversality condition does not apply here, it holds in a continuum of economies[19] associated with $\rho > 0$. Notice that in the steady state the real interest rate is $i = r + g$, and our assumption implies $i = g > 0$. Hence equations where ρ appears do not

[19] We have also checked that the equilibrium values of the endogenous variables change continuously by undertaking numerical simulations.

formally change[20]. For simplicity, we will also assume $\varphi = 0$: this eliminates the externality of basic research on applied research.

Notice that eq. (13b), the steady state definition and $r = 0$ imply:

$$v_B = \beta v_L^0. \tag{16}$$

From this and from eq.s (13a) and (13c):

$$n_A = \left(\frac{\lambda_1}{\lambda_0}\frac{1-\beta}{\beta}\right)^{\frac{1}{a}} n_B. \tag{17}$$

From equations (13d) and (13e), the steady state definition and $r = 0$ we can write:

$$v_L^0 = \left[\left(\frac{\lambda_1}{\lambda_0}\right)^{\frac{1}{a}}\left(\frac{1-\beta}{\beta}\right)^{\frac{1-a}{a}} + 1\right] v_L^1. \tag{18}$$

Imposing the steady state into (14) and using (17) yields:

Lemma 1. *The steady state equilibrium fraction of industries where basic R&D is active is*

$$m(A_0) = \frac{1}{1 + \left(\frac{\lambda_0}{\lambda_1}\right)^{\frac{1}{a}}\left(\frac{\beta}{1-\beta}\right)^{\frac{1-a}{a}}}. \tag{19}$$

Lemma 1 indicates is that the higher the difficulty of basic research (applied research), i.e. the lower λ_0 (the lower λ_1) the higher the fraction of sectors where basic (applied) R&D is needed.

This has implications for R&D enhancing regulation:

Proposition 1. *The growth maximizing upstream inventor share, β^*, of the final good patent value is equal to:*

[20] More generally, even assuming $g = 0$, and therefore $\rho = 0$ would not imply complications, as straightforward application of De L'Hospital's theorem would imply $lim_{\rho \to 0}\theta_0 = \gamma + \frac{D}{(D-Tr)w_H}$.

$$\beta^* = \frac{\lambda_1}{\lambda_0 + \lambda_1} = \frac{1}{\frac{\lambda_0}{\lambda_1} + 1}. \tag{20}$$

Proof. See Appendix.

Proposition 1 states that the innovators should be rewarded proportionally more in the stages of R&D where innovation is harder to achieve. Plugging β^* into eq. (17) implies that at the growth-maximizing policy $n_A = n_B$. Hence the innovation-maximizing share is higher in the (sub-)industries where (equilibrium) innovation is slower - expected times $\frac{1}{\lambda_0 n_B^{1-a}} > \frac{1}{\lambda_1 n_A^{1-a}}$ imply $\beta^* > 0.5$ and vice-versa.

It is important to notice that here β^* is common across industries without the risk of a "one-size-fits-all" loss (Chu, 2011) only because all industries are symmetric. However, with heterogenous industries, it would be interesting to generalize this result.

5.1 An Empirical "Rule of Thumb"

The special case we have just analyzed allows us to gain insight on evidence. In particular, from the knowledge of the basic to applied research ratio, BAR, we can easily obtain approximations for the actual values of β. In fact, eq.s (17) and (19) yield:

$$\begin{aligned} BAR &= \frac{n_B m(A_0)}{n_A \left[1 - m(A_0)\right]} = \left(\frac{\lambda_1}{\lambda_0}\frac{1-\beta}{\beta}\right)^{\frac{-1}{a}} \left(\frac{\lambda_0}{\lambda_1}\right)^{\frac{-1}{a}} \left(\frac{\beta}{1-\beta}\right)^{-\frac{1-a}{a}} \\ &= \frac{\beta}{1-\beta}, \end{aligned}$$

which implies

$$\beta = \frac{BAR}{1 + BAR}. \tag{Beta}$$

Given its extremely simple formulation, we can easily provide some of its values in the reported Table 1:

Table 1. Data for Equation (Beta)

Data on US Basic Research, US Applied Research, and US "Beta" (Upstream Bargaining Power):

Year	Basic R&D	Applied R&D	Beta
1980	16,175	25,368	0.3894
1981	16,333	27,617	0.3716
1982	16,976	29,038	0.3689
1983	18,217	31,125	0.3692
1984	19,703	33,180	0.3726
1985	21,152	36,430	0.3673
1986	24,069	38,221	0.3864
1987	25,246	38,182	0.3980
1988	26,136	39,003	0.4012
1989	27,862	41,081	0.4041
1990	28,217	42,758	0.3976
1991	32,134	45,741	0.4126
1992	31,948	43,906	0.4212
1993	32,518	42,180	0.4353
1994	32,849	40,567	0.4474
1995	32,145	44,440	0.4197
1996	34,945	45,995	0.4317
1997	38,695	48,792	0.4423
1998	36,633	48,041	0.4326
1999	39,735	53,138	0.4278
2000	42,667	56,826	0.4288
2001	46,501	63,070	0.4244
2002	49,118	48,771	0.5018
2003	51,102	57,858	0.4690
2004	51,038	64,035	0.4435
2005	52,605	62,118	0.4585
2006	52,314	65,505	0.4440
2007	55,074	69,451	0.4423
2008	56,482	72,365	0.4384

Source: *National Science Foundation* (Table A-22) and our calculations based on eq. (Beta).

Therefore, according to NSF data[21] of Table 1, in 1980 our calibrated[22] β would be equal to 38.94%, while in 2002 would have climbed up to its highest ever level of 50.18%. This value, taken on by β right in the year of the *Madey vs. Duke University* Federal Circuit's decision, exceeds 50% and is therefore clearly suboptimal, based on any positive real interest rates. After that, it has declined, including becoming 45.85% in the year of the more liberal *Merck vs Integra Lifesciences (* 2005) ruling. Of course, these calibrated values are to be taken as a "rule of thumb" indicator of the bargaining power of the upstream innovator: off-steady state and with changing interest rates they ought to be adjusted. Eq. (Beta)'s shortcut may offer a microfounded and yet easy-to-compute approximation to practitioners and policy-makers only to stimulate further investigation.

6 Blocking Patents

The present setup with basic and applied R&D is suitable to analyze a crucial issue: the patenting of basic research may hinder applied innovations because even basic concepts are patented which potentially precludes new innovations. Clearly, the incumbent monopolist in the corresponding final good sector is the natural suspect of such anti-innovative behavior. In fact, by appropriating the patent on a basic research result and stopping R&D it would eliminate expected obsolescence on its product, causing its value to jump up to $\frac{\pi}{r}$. In the steady state, the incumbent monopolist will buy the patent in order to block innovation in that sector if its willingness to pay for the research tool is higher than the outsiders' reservation price, that is if and only if:

$$v_B \leq \frac{\pi}{r} - v_L^1. \tag{21}$$

Eq. (21) is equivalent to $rv_L^1 - \pi \leq -rv_B$. This and eq. (13e) implies that

$$-\lambda_1(t)n_A(t)^{1-a}v_L^1(t) \leq -rv_B.$$

[21] Based on *National Science Foundation*, Table A-22, "Total (company, federal, and other) funds for basic research, applied research, and development, in current and constant dollars: 1953-94."

[22] See Data for Equation (Beta) in the Appendix.

Using eq. (13b), the above inequality becomes equivalent to:

$$v_B + v_L^1 \geq \beta v_L^0. \tag{22}$$

Quite interestingly, for the benchmark special case of the previous section, due to eq. (16), the inequality (22) is certainly satisfied with strict inequality. While the benchmark special case is only valid as a mathematical approximation of the steady state equilibrium equations, by continuity the following holds:

Lemma 2. *In an open right neighborhood of $r = 0$, the steady state equilibrium implies an attempt of incumbents to block innovation.*

This points to a serious problem in the patenting of basic (or applied) research, which lies in the fact that further innovation may be hindered because even basic concepts are patented which potentially precludes new innovations in that field. This is a danger, which may become true if the courts are not ready to detect such a practice[23]. In a well functional judicial system, according to Maurer and Scotchmer (2004a, p.90), courts "usually approve arrangements that remove blocking patents so that firms can bring technologies to market." The typical arrangement is compulsory licensing of the patented innovative tool. However, given the incentives to block if economically powerful incumbent patent holders highlighted in this paper, we conclude that in the presence of patentable basic research results the courts' crucial role in protecting innovative activity may become more complex.

7 Insights from Numerical Simulations

In this section we will show that when the bargaining power of the basic researcher, i.e. β, increases, it always looks bad at first. Even if it is good for long-run growth, in the short- or even medium-run it presents itself as harmful to growth. In order to show the mechanism to the reader we report two representative trajectories obtained for the endogenous variables following the announcement of a regime change in the law of motion of β. This corresponds to a sudden change in the steady state value of eq. (12) that

[23] This is an old problem in the history of patents. As reported by Scotchmer (2004, p. 14), "James Watt (d. 1819) used his patents to block high-pressure improvements... Watt's refusal to license competitors froze steam-engine technology for two decades." Fortunately, patent legal life was not as long as assumed in our model.

gradually drives the system towards the new steady state. We ran several discrete approximations of the differential equations (30), (33), (12), (36), (13b), (13d), (13e), (42),(43), (40), (41), (14), and cross-equations restrictions (13a), (13c), (9), (10), (15), and (37), obtaining remarkably robust results[24].

We set the intra-sectorial congestion parameter $a = 0.3$, consistently with Jones and Williams' (1998) and (2000) calibrations. We set the mark-up γ to 1.68, consistently with what estimated by Roeger (1995) and Martins et al. (1996). Parameter $\alpha = 0.1$ - the share of high skilled workers[25] in manufacturing production, is consistent with Berman, Bound, and Griliches, (1994). We set benchmark values of our new parameter at $\lambda_0 = \bar{\lambda}_1 = 1$, $\varphi = 0.01$, but results are robust to huge variations of them. Parameters $D = 40$, $n = 0.01$, $T_r = 4$, $\Gamma = 0.75$ follow Dinopoulos and Segerstrom (1999). For the real rate of return on consumer assets, we adopt the usual $r = 0.05$, common in the literature. As for the common law adjustment parameter, we set $\psi = 0.9$.

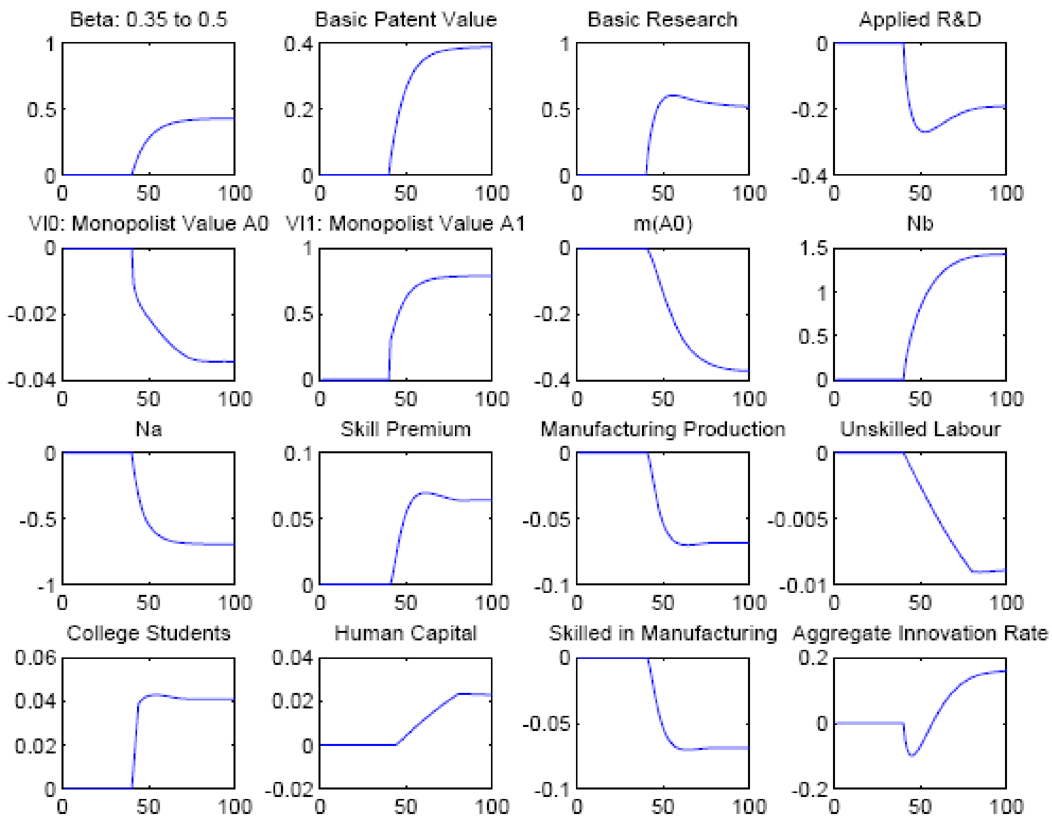
We assume that the economy begins with a steady state associated with a given value of $\bar{\beta}$. Then $\bar{\beta}$ changes and the common law share of the basic research inventor starts to head to its new steady state value.

In order to make different simulations comparable, we plot the trajectories of the deviations of the value of each variable from its initial steady state value, divided by its initial steady state value.

Figure 1 assumes that, after a long term (40 periods) initial value of $\bar{\beta} = 0.35$, it suddenly changes to $\bar{\beta} = 0.5$. As a consequence of Proposition 1, such a change will be beneficial for long term growth.

[24] We have also simulated how the dynamic behavior changes if β immediately jumps to the new steady state level, either expected or unexpected. The basic message of this section is robust to these variants.

[25] We here restrict to the share of technician workers in manufacturing in the late Eighties, as indicated by Berman, Bound, and Griliches, (1994). We are ignoring other white collars, though our simulations are quite robust to alternative specifications.



Beta: 0.35 to 0.5
Basic Patent Value
Basic Research
Applied R&D
VI0: Monopolist Value A0
VI1: Monopolist Value A1
m(A0)
Nb
Na
Skill Premium
Manufacturing Production
Unskilled Labour
College Students
Human Capital
Skilled in Manufacturing
Aggregate Innovation Rate

Figure 1

In fact, such a change is clearly growth improving from a steady state perspective: in the long run the new steady state is characterized by a higher rate of aggregate growth, a higher skill premium, a higher fraction of population choosing to educate themselves ("college students") and a higher aggregate human capital. A higher value of β means a higher fraction of the final invention appropriated by the basic researcher who invented its basic research prerequisite and a lower value of the final product appropriated by the applied researcher who invented its commerciable version. Therefore basic research is becoming more profitable (higher "Basic Patent Value", v_B) and applied research less profitable. Consequently basic research employment increases - both at the aggregate ("Basic Research") and at the industry, ("Nb") level - and applied research employment decreases both at the aggregate ("Applied R&D") and at the industry, ("Na") level. A consequence of this is that in the long run the stock market value (v_L^1) of an A_1 monopolist increases - as it faces less obsolescence - while the long run stock market value (v_L^0) of an A_0 monopolist decreases, as it faces more obsolescence. Since the positive incentives to basic R&D outweigh the negative incentives to applied R&D, R&D as a whole becomes more profitable and more skilled labour is demanded. Therefore the skill wage, w_H, increases, thereby inducing a larger fraction of the population to enrol at university. This will gradually increase the supply of human capital and decrease the supply of unskilled labour.

In the transitional dynamics, it is important to notice that as the change in the long-term court orientation $\bar{\beta}$ is forecast by the private actors, all the stock variables - $\beta(t)$, $h(t)$, $L(t)$, and $m(A_0(t))$ - are predetermined, and for example by eq. (10), $\pi(t)$ is constant. Hence only jump variables such a prices, wages, and employment change. Being $\beta(t)$ monotonically increasing, the relative incentives of basic versus applied research are gradually changed in favor of basic and to the detriment of applied research. However, the dynamics of $\beta(t)$ interacts with the intrinsically dynamic nature of the R&D process, in a way that is not captured by the mere comparative statics of steady state analysis: in fact, the expectation of higher future values of $\beta(t)$ certainly favours current basic research - the completion of which will take place in the future - without harming current applied R&D with the same intensity. In a discrete time approximation of our continuous time framework, let us imagine that basic and applied research complete - with an endogenous probability - in each period: the announcement of a higher β next period will not penalize current applied R&D, while instead it will encourage current basic research - which is promised a higher share of the future discovery. In our continuous time framework the same effect is at work: $\dot{\beta}(t) > 0$ favours the expectedly late fruits of basic research more than it reduces the expectedly earlier gains of applied research. As a consequence, aggregate R&D is favoured, and the increase in the demand for $n_B(t)$ is matched by a lower decrease in the demand for $n_A(t)$, which implies that the difference $m(A_0(t))n_B(t) - [1 - m(A_0(t))]\, n_A(t)$ increases and must be matched by a decrease in $x(t)$. The increase in the net demand for R&D labour can be satisfied only by a decrease in the manufacturing skilled-labour employment. This temporary excess demand for skilled labour is the reason for the immediate increase in the skill premium. As time passes, the increase in $w(t)$ will encourage marginally less able students to enroll to college, thereby leading to a future increase in the the aggregate supply of human capital and to a partially offsetting effect on $w(t)$. However, as long as $\beta(t)$ keeps increasing the demand for R&D labour continues to grow, though the decline in $\dot{\beta}(t)$ will eventually correct the previously mentioned intertemporal asymmetry that favoured basic research more than it disincentived applied R&D.

In the generality of simulations we have undertaken, the aggregate innovation rate decreases in an initial period following the policy switch, whose length depends on the assigned parameters: the economic reason is that R&D is shifting upstream towards basic research, thereby reducing applied R&D; this slows down the completion of existing basic research projects, which has a negative effect on innovation. However, in the case of Figure 1, in the longer run, the increase in the flow of basic research results will more than compensate a thinner applied R&D effort.

Our stylized representation suggests that policy makers should not lose their optimism about innovation enhancing policies based on shorter term R&D reallocation effects coupled with improvements in the population educational choices.

The transitional dynamics plotted in the next Figure 2 is based on the assumption that the initial value of $\bar{\beta}$ was 0.55 and it suddenly changes to 0.65. Such a change will be detrimental to long term growth, because the basic research patent owner gets entitled to too large a share of the final invention value. This discourages applied R&D too much, which more than offsets the increase in basic research. Therefore the demand for skilled labour will fall and so will the skill premium and education.

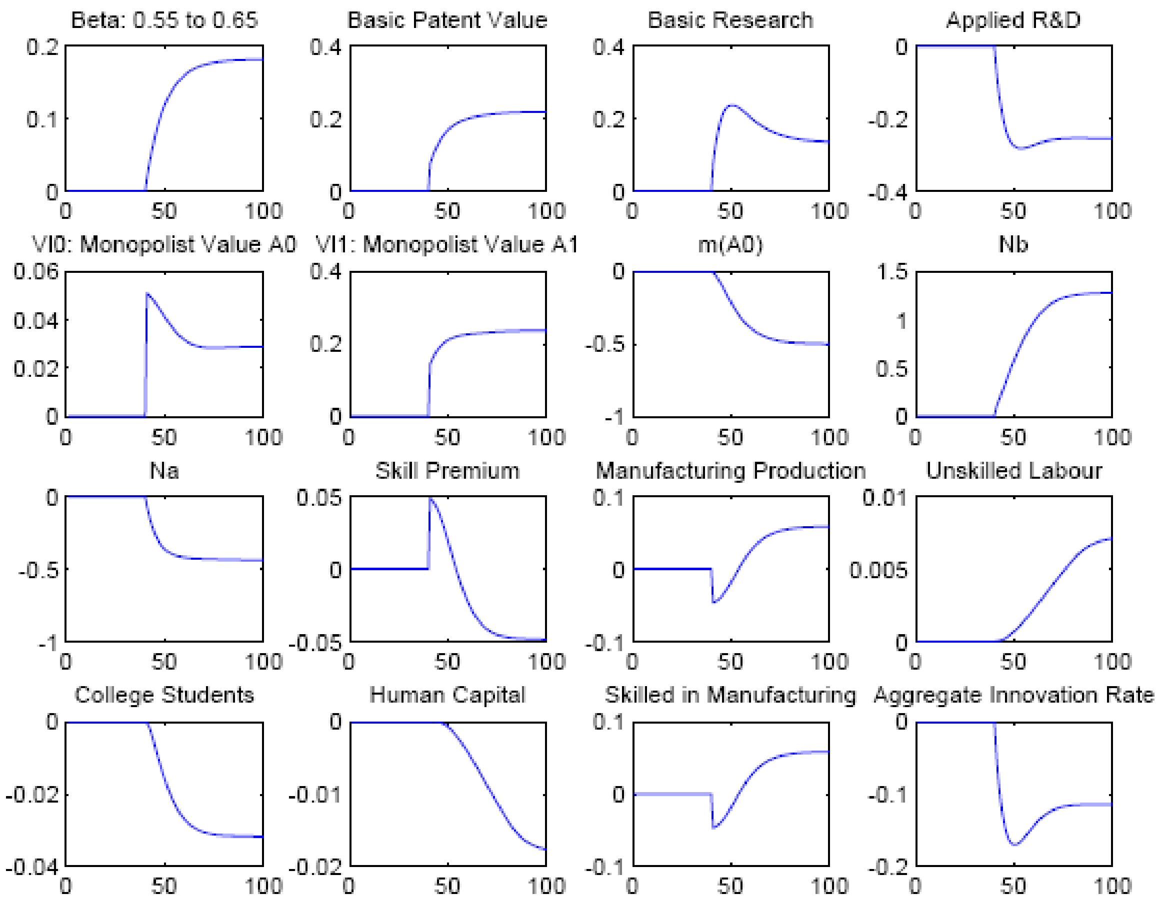

Beta: 0.55 to 0.65
Basic Patent Value
Basic Research
Applied R&D
VI0: Monopolist Value A0
VI1: Monopolist Value A1
m(A0)
Nb
Na
Skill Premium
Manufacturing Production
Unskilled Labour
College Students
Human Capital
Skilled in Manufacturing
Aggregate Innovation Rate

Figure 2

We remark that the short term reactions of the skill premium and of manufacturing production could inspire wrong interpretations of the true long term effect of normative changes. In fact, as in the previous discussion, upon impact all stock variables are given, and mainly short term announcement effects prevail. Most notably, the expected gradual increase in $\beta(t)$ fails to penalize current applied R&D in the order of magnitude as it favours current basic research: basic R&D will be entitled to a larger share of the results of *future* applied R&D, not those of current applied R&D. Such temporary win-win situation boosts aggregate R&D labour and therefore raises the skill premium. However, as $\dot{\beta}(t)$ sets in, the temporary relief for applied R&D disappears, and its smaller share of the final product patent penalizes it so much that the ensuing drop in R&D employment outweighs the increase in basic research employment - the whole effect being corroborated by the gradual increase in $1 - m(A_0(t))$ - dragging the skill premium below the initial steady state level and therefore leading towards the new steady state, characterized by less R&D employment and less innovation.

Interestingly,a gradual increase in $\beta(t)$ always leads to an immediate increase in $w(t)$: hence future strengthening basic innovation always increases the skill premium. Conversely, a gradual decrease in $\beta(t)$ always leads to an immediate decrease in $w(t)$. However, the long-term impact of these changes in $\beta(t)$ will depend on whether or not the change is growth enhancing. When it is growth enhancing $w(t)$ will eventually reach a higher steady state level, whereas the opposite holds when the long-term court orientation is detrimental to growth.

The following matrix summarizes the effects of a gradual change in $\beta(t)$ - i.e. of $\dot{\beta}(t)$ - on $w(t)$:

	Short-Run	Long-Run
Growth Enhancing $\dot{\beta} > 0$	Higher w	Higher w
Growth Harming $\dot{\beta} > 0$	Higher w	Lower w
Growth Enhancing $\dot{\beta} < 0$	Lower w	Higher w
Growth Harming $\dot{\beta} < 0$	Lower w	Lower w

As a result, our simulations warn policy against relying on empirical evaluations of IPR changes based on relatively short term effects. The short term effects of a harmful tightening, respectively relaxing, of the upstream IPR look misleadingly similar to those of a beneficial bargaining power transfer towards, respectively from, the basic researcher institutions.

The figures shown in this section are considerably robust and representative of the pro-upstream IPR changes mentioned so far: changing parameters we have observed very similar patterns of short-run and long-run dynamics[26].

8 Conclusions

The possibility that innovators may use their patents to block future innovators, and/or prevent them from commercialising their products, is a reason for concern not only among academics. The adoption by the US patent law of a statutory research exemption has been proposed as a definitive solution to this problem. But, by postponing bargaining between innovators it may put the downstream inventor at disadvantage: when is this disadvantage socially beneficial? Can we detect this from the data? This paper has tried to answer these important questions from a dynamic macroeconomic perspective.

Since the common law system implies gradual transition to new IPR regimes, we have studied the whole transitional dynamics. The most important conclusion is that the transition to a stricter regime does not appear to be monotonic, which shows how assessments based on short term data could be misleading for policy makers. For example, increases in IPR may result in a temporary reduction in economic growth, even if they can be beneficial for long-run growth.

We have also derived an easy to compute approximation of the bargaining power of the upstream researchers, offering an empirical "rule of thumb" that suggests that the year of the celebrated *Madey vs. Duke University* Federal Circuit's decision, i.e. 2005, marked the apex of the power of basic innovation patent holders, likely quite harmful for growth.

Throughout this paper, we have maintained a closed economy framework. It would be very interesting to extend our model to an open economy, whereby potentially offsetting effects may come into play, and contribute to design a more realistic picture, perhaps rendering even more complicated for policy makers to effectively gauge growth-maximizing research policy in a dynamic macroeconomy.

[26] The files used to generate them are available to the interested readers.

9 Appendix

Proof of Proposition 1. From eq. (35) and (5) follows that the steady state level of human capital per-capita is an increasing function of the skilled premium w_H, which we can write as $\overline{h}(w_H)$.

Plugging eq. (17) into the skilled labour market clearing condition (15) yields:

$$\left[m(A_0) + (1 - m(A_0)) \left(\frac{\lambda_1}{\lambda_0}\frac{1-\beta}{\beta}\right)^{\frac{1}{a}}\right] n_B = \overline{h}(w_H) - x(w_H) \equiv \Psi(w_H) \quad (23)$$

with $\Psi'(w_H) > 0$. Inserting eq. (19) into (23) we obtain:

$$\frac{n_B}{\beta\left[1 + \left(\frac{\lambda_0}{\lambda_1}\right)^{\frac{1}{a}}\left(\frac{\beta}{1-\beta}\right)^{\frac{1-a}{a}}\right]} = \overline{h}(w_H) - x(w_H) \equiv \Psi(w_H) \quad (24)$$

Plugging eq. (18) into eq. (13a) and (13e) we obtain:

$$\begin{aligned} w_H &= \lambda_0 n_B^{-a} \beta v_L^0 = \lambda_0 n_B^{-a} \beta \left[\left(\frac{\lambda_1}{\lambda_0}\right)^{\frac{1}{a}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1-a}{a}} + 1\right] v_L^1 \quad &(25a) \\ \pi &= \lambda_1 n_A^{1-a} v_L^1 = \lambda_1 \left(\frac{\lambda_1}{\lambda_0}\frac{1-\beta}{\beta}\right)^{\frac{1-a}{a}} n_B^{1-a} v_L^1 \quad &(25b) \end{aligned}$$

From the definition of profits and the steady state mass of unskilled labour, we know that $\pi = \pi(w_H)$, with $\pi'(w_H) < 0$. Dividing the last two equations side by side implies:

$$n_B \frac{1}{\beta\left[1 + \left(\frac{\lambda_0}{\lambda_1}\right)^{\frac{1}{a}}\left(\frac{\beta}{1-\beta}\right)^{\frac{1-a}{a}}\right]} = \frac{\pi(w_H)}{w_H}. \quad (26)$$

Plugging (26) into (24) gives:

$$1 = \Psi(w_H)\frac{w_H}{\pi(w_H)} \equiv \Phi(w_H) \tag{27}$$

where $\Phi'(w_H) > 0$. Therefore there exists a unique steady state level of the skill premium obtained as the solution to eq. (27). It is important to notice that, in this example, the steady state skill premium is independent of β.

The steady state innovation rate can be rewritten, after using (26), as:

$$\lambda_0 n_B^{1-a} m(A_0) = \frac{\left[\frac{\pi(w_H)}{w_H}\right]^{1-a}\beta^{1-a}}{\left[1+\left(\frac{\lambda_0}{\lambda_1}\right)^{\frac{1}{a}}\left(\frac{\beta}{1-\beta}\right)^{\frac{1-a}{a}}\right]^a} = \tag{28}$$

$$= \frac{\left[\frac{\pi(w_H)}{w_H}\right]^{1-a}}{\left[\left(\frac{1}{\lambda_0}\right)^{\frac{1}{a}}\left(\frac{1}{\beta}\right)^{\frac{1-a}{a}}+\left(\frac{1}{\lambda_1}\right)^{\frac{1}{a}}\left(\frac{1}{1-\beta}\right)^{\frac{1-a}{a}}\right]^a} \tag{29}$$

The numerator does not change with β as previously proved. The innovation rate is maximized when the denominator is minimized. Hence we need to find a value of β such that $\left(\frac{1}{\lambda_0}\right)^{\frac{1}{a}}\left(\frac{1}{\beta}\right)^{\frac{1-a}{a}}+\left(\frac{1}{\lambda_1}\right)^{\frac{1}{a}}\left(\frac{1}{1-\beta}\right)^{\frac{1-a}{a}}$ is minimized, which implies expression (20).QED.

9.1 Labour Supply and Education Dynamics

9.1.1 Unskilled Labor Supply

As previously shown, individuals born at t with ability $\theta(t) \in [0, \theta_0(t)]$ optimally choose not to educate themselves, thereby immediately joining the unskilled labour force. Hence a fraction $\theta_0(t)$ of cohort t remains unskilled their whole life. Summing up over all the older unskilled who are still alive - hence born in the time interval $[t-D, t]$ - we obtain the total stock of unskilled labour as of time t:

$$L(t) = \int_{t-D}^{t} bN(s)\theta_0(s)ds = b\int_{t-D}^{t} e^{gs}\theta_0(s)ds$$

where b is the birth rate, $N(s)$ is the population at time s.

To stationarize variables, we divide by current (time t) population e^{gs}, obtaining:

$$l(t) \equiv \frac{L(t)}{N(t)} = b\int_{t-D}^{t} e^{g(s-t)}\theta_0(s)ds.$$

Its steady state level is:

$$l = b\frac{1-e^{g(-D)}}{g}\theta_0 = \theta_0.$$

The change in the stock of the population-adjusted stock of unskilled labour is obtained by derivating $l(t)$ with respect to time:

$$\dot{l}(t) = b\theta_0(t) - be^{-gD}\theta_0(t-D) - gl(t) \tag{30}$$

As in Boucekkine et al. (2002) and Boucekkine et al. (2007) we obtain a crucial role for delayed differential equations.

9.1.2 College Population

The individuals born in t with ability $\theta(t) \in [\theta_0(t), 1]$ optimally choose to educate themselves, thereby becoming college students for a training period of duration Tr. Hence summing up over all the previous cohorts who are still in college - hence born in the time interval $[t - Tr, t]$ - we obtain the total stock of college population as of time t:

$$\widetilde{C}(t) = b\int_{t-Tr}^{t} N(s)(1-\theta_0(s))ds = b\int_{t-Tr}^{t} e^{gs}(1-\theta_0(s))ds.$$

In per-capita terms:

$$\widetilde{c}(t) \equiv \frac{\widetilde{C}(t)}{N(t)} = b\int_{t-Tr}^{t} \frac{N(s)}{N(t)}(1-\theta_0(s))ds = b\int_{t-Tr}^{t} e^{g(s-t)}(1-\theta_0(s))ds. \tag{31}$$

In a steady state:

$$\widetilde{c} = b\frac{1-e^{g(-Tr)}}{g}(1-\theta_0). \tag{32}$$

Taking the derivative of eq. (31) with respect to time we obtain:

$$\dot{\widetilde{c}}(t) = b\left(1 - \theta_0(t)\right) - be^{-gTr}\left(1 - \theta_0(t - D)\right) - g\widetilde{c}(t). \tag{33}$$

9.1.3 Human Capital

The stock of skilled workers will coincide with those students who have completed their education and are still alive, born in $[t - D, t - Tr]$:

$$\widetilde{H}(t) = b\int_{t-D}^{t-Tr} N(s)(1 - \theta_0(s))ds = bN(t)\int_{t-D}^{t-Tr} e^{g(s-t)}(1 - \theta_0(s))ds \quad (3)$$

The total workforce (including students) in equilibrium equals total population, hence:

$$L(t) + \widetilde{H}(t) + C(t) = e^{gt}.$$

Due to heterogeneous learning abilities, in order to obtain the aggregate skilled labour supply, we need to multiply each skilled worker by the average amount of human capital that she can supply, given by the average skill of her cohort net of dispersion parameter Γ:

$$\int_{\theta_0(t)}^{1} (\theta - \Gamma)\frac{1}{1 - \theta_0(t)}d\theta = \frac{1 + \theta_0(t) - 2\Gamma}{2}.$$

Therefore the aggregate amount of skilled labour in efficiency units (skilled labor supply) is:

$$H(t) = bN(t)\int_{t-D}^{t-Tr} \frac{e^{g(s-t)}(1 - \theta_0(s))\left(1 + \theta_0(s) - 2\Gamma\right)}{2}ds$$

Dividing by time t population, we can express per-capita human capital as:

$$h(t) \equiv \frac{H(t)}{N(t)} = \frac{b}{2}\int_{t-D}^{t-Tr} e^{g(s-t)}(1 - \theta_0(s))\left(1 + \theta_0(s) - 2\Gamma\right)ds. \tag{34}$$

The steady state value is:

$$h = b\frac{\left[e^{g(-Tr)} - e^{g(-D)}\right](1-\theta_0)(1+\theta_0-2\Gamma)}{2g} \tag{35}$$

The dynamics of human capital can be studied by derivating this expression with respect to time:

$$\begin{aligned} \dot{h}(t) &= -gh(t) + \frac{b}{2}e^{-gTr}(1-\theta_0(t-Tr))(1+\theta_0(t-Tr)-2\Gamma) - \\ &\quad + \frac{b}{2}e^{-gD}(1-\theta_0(t-D))(1+\theta_0(t-D)-2\Gamma). \end{aligned} \tag{36}$$

9.2 Transitional Properties of Educational Choice

The study of the transition dynamics of this model is complicated by the skilled/unskilled labour dynamics and by the endogenous education choice under perfect foresight. Key to the solution is the transformation of the integral equation for the ability threshold level for education into a set of differential equations.

Defining the present value of the unskilled wage incomes as $W_U(t) = \int_t^{t+D} e^{-\int_t^s i(\tau)d\tau} ds$ and the present value of the skilled wage income as $W_S(t) = \int_{t+Tr}^{t+D} e^{-\int_t^s i(\tau)d\tau} w_H(s)ds$, we know from (4) that

$$\theta_0(t) = \Gamma + \frac{W_U(t)}{W_S(t)}. \tag{37}$$

Defining

$$\begin{aligned} R_1(t) &= e^{-\int_t^{t+D} i(\tau)d\tau}, \text{ and} && (38) \\ R_2(t) &= e^{-\int_t^{t+Tr} i(\tau)d\tau} && (39) \end{aligned}$$

we can write:

$$\begin{aligned} \dot{W}_U(t) &= R_1(t) - 1 + i(t)W_U(t) && (40) \\ \dot{W}_S(t) &= R_1(t)w_H(t+D) - R_2(t)w_H(t+Tr) + i(t)W_S(t). && (41) \end{aligned}$$

Differentiating eq.s (38)-(39) with respect to time we obtain:

$$\begin{aligned} \dot{R}_1(t) &= R_1(t)(i(t) - i(t+D)), \text{ and} && (42) \\ \dot{R}_2(t) &= R_2(t)(i(t) - i(t+Tr)). && (43) \end{aligned}$$

These equations allow us to cast our model in a framework that can be studied in terms of delayed differential equations.

9.3 Expenditure and Manufacturing Dynamics

From eq.s (10) follows:

$$\frac{\gamma - 1}{\gamma} E(t) = (\gamma - 1) \frac{1}{1-\alpha} l(t). \tag{44}$$

Log-differentiating with respect to time, using Euler equation (3) and the unskilled law of motion (30) yield:

$$i(t) - (\rho + g) = \frac{\dot{E}(t)}{E(t)} = \frac{\dot{l}(t)}{l(t)} = \frac{b\theta_0(t) - be^{-gD}\theta_0(t-D)}{l(t)} - g \tag{45}$$

that - since $r(t) = i(t) - g$ - can be rewritten as

$$r(t) - \rho = \frac{b\theta_0(t) - be^{-gD}\theta_0(t-D)}{l(t)} - g, \tag{46}$$

In the steady state: $r(t) = \rho$.

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